Turbulent Swirling Flame Prediction

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Finite-difference predictions are made of turbulent boundary layer swirling flames. This paper is concerned essentially with a more accurate simulation of the flow, the turbulence and reaction models. Current developments are discussed. It is demonstrated that the effect of swirl on flame size, shape, and combustion intensity may be represented by nonisotropic mixing-length and energy-length turbulence models and an eddy-break-up turbulence-controlled reaction model.

Nomenclature

A = axial velocity decay parameter
a = apparent origin distance
B = swirl velocity decay parameter
C = constant
c = weighted mean specific heat
d = nozzle diameter
G = axial flux of momentum
g = mean square fluctuating component of fuel concentration, $\overline{m_{fu}'^2}$
H = heat of combustion
h = stagnation enthalpy
i = stoichiometric ratio
J = turbulent flux tensor
k = kinetic energy of turbulence
k_u = axial velocity error curve parameter
l = Prandtl mixing length
ℓ = length scale of turbulence
m = chemical mass fraction
p = time-mean pressure
R = universal gas constant, mass rate of creation per unit volume (with subscript)
Ri = Richardson number
S = swirl number, $G_\theta/(G_x d/2)$, source term (with subscript)
T = time-mean temperature
t = time
u, v, w = time-mean axial, radial and swirl velocities
W = mean square vorticity fluctuation
x, r, θ = axial, radial, and polar coordinates
α = jet half-angle ($u/u_m = 0.5$)
ε = turbulence energy dissipation rate
λ = mixing length parameter
μ = turbulent viscosity
ξ = nondimensional radial coordinate, $r/(x+a)$
ρ = time-mean density
σ = Prandtl-Schmidt number
ϕ = general dependent variable

Superscripts
$(\ )'$ = turbulent fluctuating component
$(\bar{\ })$ = time-average
$(\dot{\ })$ = flow rate

Subscripts
EBU = relating to eddy-break-up model
fu, ox, pr = relating to fuel, oxidant and product (including inerts)
$g1, g2$ = relating to constants in g equation
o = value at orifice of jet
p = relating to constant pressure
R = relating to constant in k/ℓ equation
rx, etc. = rx-component of second order tensor, etc.
x = relating to value at particular axial station
x, θ = relating to directions x, θ
ϕ = relating to general dependent variable ϕ
0.01, etc. = relating to position where $u/u_m = 0.01$, etc.

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Index categories: Boundary Layers and Convective Heat Transfer—Turbulent; Jets, Wakes, and Viscid-Inviscid Flow Interactions; Reactive Flows.

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I. Introduction

SWIRL is used extensively in combustion chambers and industrial furnaces as a means of controlling flame size, shape, stability, and combustion intensity. Previous experimental studies[1–9] characterize these large-scale effects and good reviews of these studies are available.[10–12] The more convenient computational solution of such flow is of considerable interest but until recently presented formidable simulation and solution problems. The swirl strength determines the degree of upstream influence and the swirl number S, being a nondimensional number representing the axial flux of angular momentum divided by the axial flux of axial momentum times nozzle radius, measures this degree of swirl. A strongly swirling flame (approximately $S \geqq 0.6$) possesses strong radial and axial pressure gradients in the region of the orifice; the axial one is sufficient to cause the interesting and useful effect of a recirculation zone to be set up. Weaker swirl does not cause such strong curvature of the streamlines as to result in recirculation, and a boundary-layer jet flame results. This paper is concerned with a direct finite-difference solution to flows of the latter type and an investigation to find suitable turbulence and reaction models.

The technique provides an illustration of current predictive capability and offers a contribution toward the development of turbulence and reaction models for these flows. Recent progress in numerical solution procedures has ensured that the governing equation system may be solved fairly readily.[13] The author has already reported the basic numerical finite-difference prediction procedure for swirl flows.[14,15] The technique is stable, accurate and economical in its solution of the equations. For nonreacting flows, predictions have been made with nonisotropic-mixing-length and energy-length turbulence models which simulate well the effects of swirl.[15] Previous calculations with the simple mean-value Arrhenius reaction model predict only qualitatively the flame zone and the effects of swirl.[14] Similar computational works[16,17,20] have omitted the inclusion of current ideas on the simulation of turbulent reacting flow. This paper is concerned

essentially with a more accurate simulation of the flow, the turbulence and reaction models.

A brief recollection is made of the technique for solving the system of equations governing multi-component boundary-layer flow with turbulence, swirl, and chemical reaction. Incorporation is made of the more advanced turbulence models with nonisotropic variants and an eddy-break-up turbulence-combustion model. Solutions are compared with previous experimental data and appropriate values of parameters are deduced in order that flows of this type may now be satisfactorily predicted.

II. Basic Equations and Computational Procedure

The Simple Chemically-reacting System

Consideration is given to a simplified exothermic chemical reaction of the form[13,18]

$$1 \text{ kg fuel} + i \text{ kg oxidant} \rightarrow 1 + i \text{ kg product} + H_{fu} \quad (1)$$

where combustion is assumed to occur in a single step between just two species, fuel and oxidant. All c_p's are taken as independent of temperature; all transport properties (exchange coefficients) for stagnation enthalpy, fuel, oxidant, and product are taken to be equal but may vary together from point to point in the flow. With these assumptions H_{fu} is independent of T and h, T, m_{fu} and u are related via

$$h = c_p T + H_{fu} m_{fu} + u^2/2 \quad (2)$$

Note also that

$$m_{fu} + m_{ox} + m_{pr} = 1 \quad (3)$$

at any instant at any point. Also with these assumptions $m_{ox} - i m_{fu}$ becomes a conserved property obeying a standard differential equation with zero source. These equations are supplemented by an ideal equation of state which uses a weighted mean mixture molecular weight.

Basic Equations

Parabolic partial differential equations of the form

$$\rho(u\,\partial\phi/\partial x + v\,\partial\phi/\partial r) = (1/r)\,\partial(rJ_\phi)/\partial r + S_\phi \quad (4)$$

where

$$J_\phi = (\mu_{rx}/\sigma_\phi)\,\partial\phi/\partial r$$

for ϕ equal to u, rw, k, and $k\ell$ have been developed for isothermal turbulent boundary-layer swirling flows. These equations differ primarily in their final source terms. For u, it is the longitudinal pressure gradient due to swirl; for rw, it is zero after the diffusion terms has been substantially modified; for k, it consists of terms for the rate of generation of energy by shear stresses and the rate of dissipation by viscous action; and for $k\ell$ there are similar terms. Full details are in a previous paper.[15]

For a reacting flow, this set of equations is supplemented by similar equations for ϕ equal to h, m_{fu}, $m_{ox} - i m_{fu}$ and g. Again these differ primarily in their final source terms. For h, it is the kinetic heating terms; for m_{fu}, it is R_{fu}, the mass rate of creation of fuel per unit volume (negative); for $m_{ox} - i m_{fu}$, it is zero; and for g, it consists of terms for the generation and dissipation of g being specifically

$$C_{g1}\,\mu_{rx}(\partial m_{fu}/\partial r)^2 - C_{g2}\,\rho g k^{1/2}/\ell \quad (5)$$

If one does not wish to solve the differential equation for g, it may be obtained approximately by assuming that generation of g equals dissipation of g locally. In this case it suffices to use the algebraic equation

$$g = (C_{g1}/C_{g2})\ell^2(\partial m_{fu}/\partial r)^2 \quad (6)$$

Full details are in the literature.[18,19]

Prior to solution of the basic equations two further equations are required. One is the usual continuity equation and the other is the radial pressure gradient due to swirl

$$\rho w^2/r = \partial p/\partial r \quad (7)$$

Gross transport integrals across the mixing layer lead to[14]

$$\left.\begin{aligned} G_x &= \int_0^\infty (\rho u^2 + p - p_\infty) r\,dr \\ G_\theta &= \int_0^\infty \rho u w r^2\,dr \\ \dot{m}_{ox} - i\dot{m}_{fu} &= \int_0^\infty \rho u (m_{ox} - i m_{fu}) r\,dr \\ \dot{m} &= \int_0^\infty \rho u r\,dr \\ \dot{m}_{fu} &= \int_0^\infty \rho u m_{fu} r\,dr \end{aligned}\right\} \quad (8)$$

The first three are independent of axial distance x; they represent the axial fluxes of axial momentum, angular momentum, and $m_{ox} - i m_{fu}$. The mass flow rate $\dot{m}$ increases with x as the flow entrains mass; the fuel flow rate $\dot{m}_{fu}$ decreases with x as chemical reaction proceeds.

A local nondimensional swirl number[14,15] may be taken as $S_x = G_\theta/(G_x r_{0.01})$ and at any x it characterizes the effect of rotation on the flow. In a free swirling jet it decreases with x as the jet spreads from its initial value of $S = G_\theta/(G_x d/2)$ which is called the swirl number of the jet.

Computational Procedure

The eight partial differential equations for ϕ equal to u, rw, k, $k\ell$, h, m_{fu}, $m_{ox} - i m_{fu}$, and g are solved via an advanced version of the fully implicit Patankar-Spalding GENMIX finite-difference procedure. The technique is stable, accurate, and economical and has been extensively tested. Global accuracy for G_x, G_θ, and $\dot{m}_{ox} - i\dot{m}_{fu}$ is exceptionally good. All the details are well documented elsewhere.[13,14,20]

III. Turbulence and Reaction Models

Turbulence Models and the Effect of Swirl

Closure of the time-mean equation system is effected by means of a turbulence model and models are generally classified according to the turbulent flux hypothesis (whether or not turbulent exchange coefficients are introduced) and the number of extra differential equations to be solved.[21] If introduced, exchange coefficients have generally been assumed isotropic until recently, even in swirling flows. Appropriate models to effect closure for isothermal swirling flows have been discussed in previous work.[15] Mixing-length and energy-length models are examples of theories of the exchange coefficient type and both these have been modified to account for the effect of swirl on the primary viscosity component μ_{rx} and the nonisotropic nature of the turbulent viscosity in flows with swirl.[15,22-24] For boundary-layer swirl flows, interest centers on the specification of μ_{rx} and $\mu_{r\theta}$ (or equivalently $\sigma_{r\theta}$). Briefly the models are:

Prandtl mixing length model (PML)

μ_{rx} is taken as proportional to the second invariant of the mean flow rate of deformation tensor and nonisotropy is obtained by use of a variable $\sigma_{r\theta}$. Thus

$$\left.\begin{aligned} &\mu_{rx} = \rho l^2\{(\partial u/\partial r)^2 + [r\,\partial(w/r)/\partial r]^2\}^{1/2} \\ &l = \lambda r_{0.05} \\ &\lambda = 0.08\,(1 + \lambda_s S_x) \\ &\lambda_s = \text{const} \\ &\sigma_{r\theta} = \text{const dependent on } S \text{ or variable dependent on } S_x \end{aligned}\right\} \quad (9)$$

The differential equations for k and $k\ell$ are not used.

Energy-length model ($k - k\ell$)

The relation

$$\mu_{rx} = \rho k^{1/2}\ell \quad (10)$$

is used and differential equations for k and $k\ell$ are solved. These equations are of the standard type and the effect of swirl is represented via an additional source term $C_R\,\rho R i k^{1.5}$ in the $k\ell$ equation. A characterization of the form $C_R = \text{const}$, and

$\sigma_{r\theta}$ = const dependent on S or variable dependent on S_x, was found suitable for isothermal flows. Schmidt numbers for k and $k\ell$ are taken as unity.

These two models are retained for the flame considered here. Other turbulent exchange coefficients are related to μ_{rx} via Prandtl-Schmidt numbers and these are all taken as 0.7.

Note that though the author solves for the variable $k\ell$, others solve for different combinations of k and ℓ. For example, equations for $\varepsilon = k^{1.5}/\ell$ and $W = k/\ell^2$ have been developed and, whereas there is little difference in the results between these for boundary-layer flows, the indications are that for general flows the ε equation is preferable.[25]

Note also that the PML model is a degenerate form of the k-$k\ell$ model in which convection and diffusion of k are neglected. In this case

$$\left.\begin{aligned} k &= l^2\{(\partial u/\partial r)^2 + [r\,\partial(w/r)/\partial r]^2\}/C_D^{1/2} \\ \ell &= lC_D^{1/4} \end{aligned}\right\} \qquad (11)$$

Reaction Models

The combustion process is generally taken to be represented by the previously described single exothermic chemical reaction between just two species combining to form a third. The reaction rates may be specified by naive expressions (for turbulent flows) of the Arrhenius type. Errors are generated because in reality many intermediate compounds occur and time-average values are used in highly nonlinear functions. Often eddy-break-up overrides chemical influence. The latter difficulty is most serious and a turbulence-controlled reaction model for coping with this has been suggested by Spalding, using an eddy-break-up k-W-g model.[26]

The Arrhenius model (naive for turbulent flows)

Reaction rates are specified via simple time-averaging of the instantaneous Arrhenius expression, so

$$R_{fu} = -Pp^2 m_{fu} m_{ox} \exp(-E/RT) \qquad (12)$$

where time-mean values are used on the right. The equation for g is not required. The model is rightly called naive because it is known that the fluctuations in temperature and concentrations can be extremely large and that the instantaneous reaction rate function is highly nonlinear. This suggests that simple time-averaging will not be satisfactory. Moreover, if it is supposed that the mixture is mainly composed of fragments of fuel, oxidant, and product, the availability of fuel for burning is strongly related to the turbulence structure. A model which promotes the effect of turbulence on the reaction rate and reduces the chemical-kinetic influence would seem to be more realistic. Such a model has been devised by Spalding[18,26] and will now be described.

The Eddy-Break-Up model (EBU)

The model involves the calculation of g, the mean square fuel concentration fluctuating component, either from its differential equation or its approximate algebraic form Eq. (6). The fluctuations of temperature and oxygen concentration are supposed to be perfectly correlated with the fuel fluctuations. Hence a chemical-kinetic limit is set to the reaction rate but a central point of the model is that there is a second limit, often much lower, set by the rate of dissipation of the fluctuations. This is a process which involves break-up of large eddies into small ones and though molecular processes are involved in the final stages of dissipation they are not the controlling ones. It is the fuel concentration dissipation rate which controls the reaction rate, and Spalding[18] has proposed the eddy-break-up reaction rate expression

$$R_{fu} = -C_{EBU}\,\rho g^{1/2} k^{1/2}/\ell \qquad (13)$$

for a turbulent flame of high temperature, where C_{EBU} is a universal constant for high Reynolds number flow. When both the chemical-kinetic and eddy-break-up limits are of the same order of magnitude the lower value is used, but more recent work indicates that the eddy-break-up expression for the reaction rate is adequate on its own.[27] Indications are that the value of C_{EBU} is 0.53. Note that this reaction model may be used with either turbulence model, via the use of Eq. (11).

IV. Results and Discussion

The results presented and discussed refer to predictions made for turbulent swirling flames studied experimentally by Chigier and Chervinsky.[2,4] The flames were premixed ($m_{fu} = 0.245$) and the initial fuel/air ratio was well outside the flammability limits. Velocities and velocity gradients were too large to permit flame stabilization at the burner rim and it was some 4–6 diameters downstream that they were sufficiently reduced to allow an annular flame front to be stabilized in the jet boundaries. The main reaction zone was confined to the annular space between the cool central core and the cold surrounding air, temperatures rising in the core in the downstream direction largely due to turbulent mixing with hot combustion gases from the reaction zone.

The Simulation

Computations begin at the burner exit with a cold premixed jet. Initial velocities are between 60 and 100 m/sec and chemical reaction is allowed to proceed only at points where the velocity is below 24 m/sec, in common with experimental data.[2] Both the PML and k-$k\ell$ turbulence models with nonisotropic allowance are considered but only results with the eddy-break-up reaction model are presented. C_{EBU}, C_{g1}, and C_{g2} are taken as 0.53, 3.0, and 0.132.

Parametrization

The PML model has been used to generate solutions. By individually varying λ_s and $\sigma_{r\theta}$ the effect of each on the flow may be deduced. Increasing λ_s leads to a progressive downward trend of u_m and w_m; increasing $\sigma_{r\theta}$ has little effect on u_m but provides a less rapid decay of w_m. Consideration of these initial results enables a characterization of swirl to be deduced. Since experimental results on w_m values show some spread, it is not possible to deduce definite nonisotropic turbulence in these flows. The form found appropriate in isothermal flows is used throughout, for the greatest effect of swirl is in the initial region before the flame begins. The recommended parameters are

$$\lambda_s = 1.2 \qquad \sigma_{r\theta} = 1 + 5S_x^{1/3} \qquad (14)$$

and longitudinal decays with this characterization are shown in Fig. 1. The decays compare well with experimental results and the progressive increase in the u_m and w_m decays, as S increases, are clearly evident. It is to be noted, however, that the velocity decays are slower than in cold swirling jets. This is largely due to the temperature and density changes, and a consequence of this gas expansion is increased axial and radial velocities, giving a reduced rate of decay of u_m and a wider jet initially.

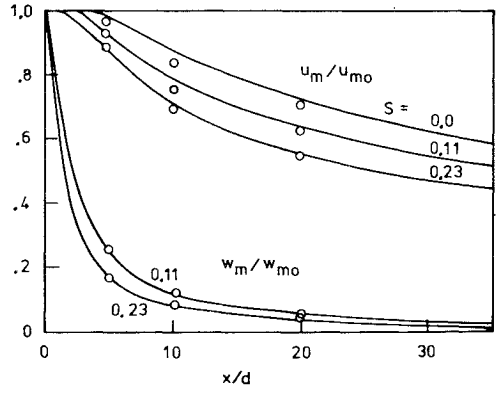

Fig. 1 Longitudinal velocity decays with PML model (0 = experimental data).

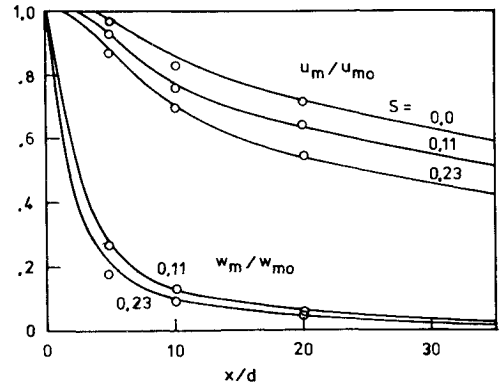

Fig. 2 Longitudinal velocity decays with k-$k\ell$ model (0 = experimental data).

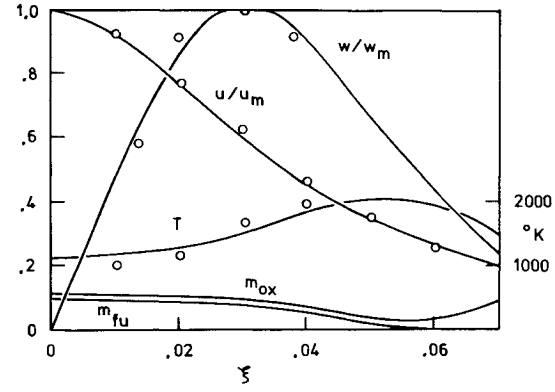

Fig. 4 Lateral profiles for $S = 0.16$ at $x/d = 10$ (0 = experimental data).

The predictions were repeated using the k-$k\ell$ model. Parameters or functions C_R and $\sigma_{r\theta}$ supposedly allow the effect of swirl to be catered for. The longitudinal effect of increasing the former is to produce a progressive downward trend of both u_m and w_m; increasing the latter has some effect on the u_m decays but its greatest effect is to produce less rapid decay of w_m. Again it is not possible to deduce definite nonisotropic turbulence in these flows; the form found appropriate for isothermal flows is retained, computer optimization yields the values

$$C_R = 0.15 \qquad \sigma_{r\theta} = 1 + 2S_x^{1/3} \tag{15}$$

and longitudinal decays are shown in Fig. 2. These are very similar, as may be expected, to the ones obtained with the PML model and again compare well with experimental results.

General Features

Subsequent results are restricted to those obtained using the k-$k\ell$ model, those with the PML model being very similar. Figure 3 shows longitudinal variations with swirl of T_c, the centerline temperature, and $\dot{m}_{fu}$, the total mass flow rate of unburned fuel. Notice that the centerline temperature increases more rapidly as S increases, indicating the more rapid mixing of hot combustion products with the cooler higher velocity core region. Observe also that $\dot{m}_{fu}$ decreases more rapidly as S increases, indicating the more rapid consumption of fuel per unit length of flame.

Lateral profiles are shown in Fig. 4 for the $S = 0.16$ flow at the axial station $x/d = 10$. The $\xi = r/(x+a)$ values used for the abscissa are a little deceptive, since a/d increases with swirl from 35 for $S = 0.0$ to 60 for $S = 0.23$. This is a consequence of the sudden expansion at the flame front and, although profile variations with S are not very significant when plotted against ξ, there is a progressive increase in jet width as S increases. The u profile has changed from the near plug flow initial one to a more gaussian one; the w profile from a near solid body rotation to a Rankine free-forced vortex type. The fuel and oxygen concentrations have significantly altered—much of the oxygen in the initial jet has been burned or diluted with combustion product. Diffusion and lack of combustion in the cool higher velocity core has allowed much overlapping of the fuel and oxygen concentration profiles. Reaction is especially intense in the region near the edge of the jet where velocities are low and temperature maxima are found here and not on the jet axis.

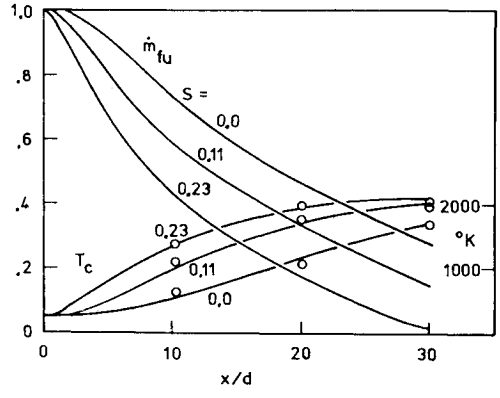

Fig. 3 Longitudinal T_c and $\dot{m}_{fu}$ variations with k-$k\ell$ model (0 = experimental data).

The temperature in the cool high velocity central core increases as x increases due to mixing with hot combustion products. Full similarity is obtained further downstream where the main reaction zone and position of T_m converge onto the jet axis. Increased x or S would produce more fully developed profiles and vice versa. Then the u profile would be even more gaussian, the cool core would be hotter, and T_m would be nearer to the centerline and gaussian near the outer edge. The fuel concentration would have reduced and the oxygen concentration increased. The concentration profiles would be less overlapped, the reaction diffusion controlled, and the position of minimum oxygen level and maximum reaction would have approached the axis. Maximum and minimum temperatures at points in the flow are not shown—these are deduced from the fuel concentration fluctuations and show the large variations in T, m_{fu} and m_{ox} at points in the turbulent flow. Temperature fluctuations of the order of several hundred degrees appear, most prominently in the main reaction zone. Even further downstream where the reaction is diffusion controlled the value of the g equation is that it still gives fluctuations in m_{fu}, m_{ox}, and T, so allowing nonzero time-average concentrations and a thick flame region.

Temperature field predictions clarify the effect of swirl on flame size, shape, and combustion intensity. Figure 5 shows the flame front lines (loci of temperature maxima) and it is clear that the length of the flame decreases markedly with swirl and that there is a progressive increase in the initial width (at $x/d = 10$) of the flame as S increases. Flame lengths as determined from Fig. 5 may be compared favourably with experimental values as shown in Table 1.

Table 1 Flame lengths

S	0.0	0.11	0.16	0.23
Predicted length	43	38	30	23
Experimental length	...	40	31	20

Trends

The results show that the effect of swirl on jet flame development may be predicted with either of the two turbulence models.

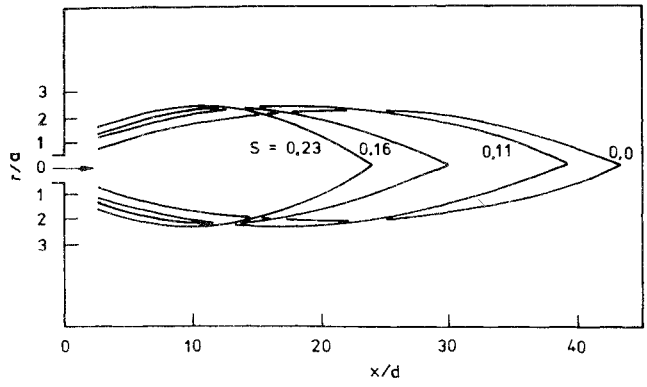

Fig. 5 Flame front lines.

The primary use of swirl in a jet flame is to increase the flame width and mass entrainment, quicken the decay of axial velocity, and decrease the flame length. The swirl velocity field is of secondary importance to the effect of the initial degree of swirl on the subsequent flow. This is of considerable interest in engineering applications and for practical purposes it is necessary to know the variation with swirl of parameters concerned with flame development. Jet development parameters A (for axial velocity decay), B (for swirl velocity decay), k_u (axial velocity error curve parameter), and α (jet half angle) are defined in terms of a (which increases with swirl) by

$$\left.\begin{aligned} u_m/u_{mo} &= A(\rho_\infty/\rho_m)^{1/2}\, d/(x+a) \\ w_m/w_{mo} &= B(\rho_\infty/\rho_m)^{1/2}[d/(x+a)]^2 \\ u/u_m &= \exp(-k_u \xi^2) \\ \tan\alpha &= r_{0.5}/(x+a) = (\ln 2/k_u)^{1/2} \end{aligned}\right\} \quad (16)$$

Recommendations for the range $x/d = 10$–20 from predicted values which match experimental data are given in Table 2.

Table 2 Jet development parameters

S	0.0	0.11	0.23
a/d	35	45	60
A	15	16	17
B	—	100	125
k_u	470	470	430
α	2.2°	2.2°	2.3°
Flame Length/d	43	38	23

V. Conclusions

A finite-difference solution procedure now exists for boundary-layer turbulent swirling flames. It is demonstrated that the effects of swirl on flame size, shape, and combustion intensity may be represented by nonisotropic mixing- and energy-length turbulence models and an eddy-break-up turbulence-controlled reaction model. Recommended parameters for the turbulence models are given in Eqs. (14) and (15), where the local swirl number and Richardson number play important roles in linking the $r\theta$-shear with the rx-viscosity. The eddy-break-up reaction model shows considerable promise and its further development appears to be one of the most urgent tasks in combustion theory.

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